On the

Spectral Analysis of Sodium and LED Dispersion

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Introduction

Interferometry is a highly precise measurement technique which allows for the probing of otherwise inaccessible regions of the world around us. To this end, the analysis of small scale features such as spectral characteristics is perfectly suited to the use of interferometer techniques. The close spacing of sodium emission lines provides an ideal means to test the limits of the Michelson Interferometers accuracy. At first glance a light source may appear to emit a very specific frequency of light however commonly this is not the case and provides yet another ideal means by which the boundaries of interferometer precision can be analysed. The distribution of emitted light and spacing of spectral lines provide an ideal means to quantify the extent of the interferometers measurement capability and provides seed for further investigation.

Optimisation of apparatus

Due to the precise nature of measurements using light interference, optimum apparatus set-up is crucial in receiving the desired results. A Michelson interferometer was constructed as displayed in diagram 1.

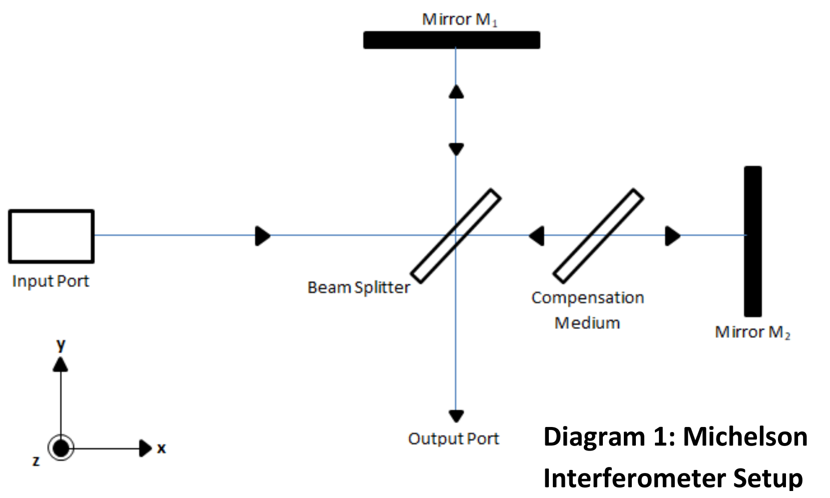

Diagram 1: Michelson Interferometer Setup

Where mirror M_2 was able to be rotated in the z-axis (defined in diagram 1) about its centre and mirror M_1 was able to be adjusted vertically in the y direction. The adjustability of these mirrors allowed for altercation in the optical distances travelled by light along both arms of the interferometer proving a crucial part of the interference process. Rotation of mirror M2 creates a rotational interference condition while motion of mirror M_1 provides a translational interference condition (Ciddor, P.E 1973). The central beam splitter acts to split the input port ray towards the two mirrors. Rays reflecting from mirror M_1 will pass through the beam splitter two additional times relative to the rays incident on mirror M_2. A compensation medium is placed along the second ray's path in order to compensate for this fact and make the distance travelled by both beams equal. To ensure beams travelling along both the vertical and horizontal arms correctly align upon recombination at the observation port, an incandescent lamp was utilized. Upon placing the lamp at the input port the constructed image was observed at the output port. Here two images were seen, one generated by each arm length. To ensure both mirrors M_1 and M_2 were aligned to be perpendicular, mirror M_2 was rotated until both images were closely superimposed. This minimises the rotational interference condition.

To ensure the translational interference condition is also minimised, caused by differences in arm length, a mercury lamp was made use of. Care was taken that a diffusing glass screen was placed in front of the lamp at all times to block the emission of UV rays which can prove harmful to the eye if observed. An additional green filter was positioned afore the lamp, to further filter the emitted wavelengths. Mirror M1 was then adjusted along the vertical axis while the image produced was observed at the output port. Adjustment was continued until the interference fringes produced at the output were as straight as possible (parallel to the z-axis) and of maximum width. This condition corresponds to the offset between the two arm lengths being minimised (Ciddor, P.E 1973).

With the aforementioned conditions satisfied the apparatus is then in a state from which further analysis of different spectral features can be investigated, knowing that zero offset caused by mirror orientations has been negated.

Lever Ratio Method

The relationship between changes in arm length, adjustable by the micrometre wheel which acts to translate mirror M_1, and the passing of fringes is defined by the following relationship:

$$2|R_2 - R_1|K = m\lambda$$

Where R_1 and R_2 are micrometre scale readings before and after adjustment respectively, m is the number of fringes passed at the observing port, λ is the wavelength of the light and K is a constant of proportionality. For a derivation of this equation see appendix section 1:A.

Knowing the wavelength of mercury light in the green region to be 546.1nm (G Hill, 2018) by adjusting the micrometre head and measuring the passage of m fringes the proportionality constant was able to be inferred. To provide a large enough sample size the passage of ten fringes was chosen and recorded for various adjustments of the micrometre head. Following 9 such measurements the sample size was increased to 20 fringes to further reduce the spread of results and allow a K value to be more accurately determined. 6 such measurements were taken and the corresponding K value then calculated. The standard deviation of the resulting value was then computed and divided by $\sqrt{n}$ for n the number of data points, to give the standard deviation of the calculated mean from the actual mean (Calkins, K 2005).

Results for apparatus optimisation

The recorded lever arm ratio following experimental measurement was determined to be:

$$k = 0.195 \pm 0.001$$

Where uncertainty was limited by the minimal length measureable by the micrometre head, in this case 0.0025 mm. Standard deviation of the measured value was 0.0044 while the standard deviation of the calculated mean from the theoretical mean, provided by $\sqrt{n}$ theory (Calkins, K 2005), was 0.56. For a list of all measured and calculated data, including uncertainty derivations, see appendix section 1:B.

Discussion of results for apparatus optimisation

The calculated value of the lever arm ratio is accurate to a significant level and falls within the necessary bounds for standard deviation specified by the practical outline (Hill, G 2018). This allows for the ensuing use of the lever arm relation in converting shifts in micrometre position to the passing of fringes at the observing port. Initially only 10 fringe passes were recorded however it was noted the running value of the SD was consistently above the desired level. Due to this 20 fringe passes were instead measured to increase the sample size and provide a more accurate measure of the K value.

Spectral Gap between Sodium Lines

Provided the distance between two spectral wavelengths is relatively small, it is possible to have an optical path difference such that the constructive interference points of one wavelength at the observation port align almost perfectly with the destructive interference points of the second wavelength (Hill, G 2018). This phenomenon has the effect of minimising the visibility of the observed pattern at the output port. Given a slight change in OPD it is possible to have inter-wavelength constructive and destructive coincidence once more leading to a point of maximum visibility. This pattern then repeats and a relationship between visibility and OPD emerges.

This effect is prominently noticeable in Sodium light which features two very closely spaced wavelengths in the 580-590nm region (Hyperphysics 2018). The exact spacing of these wavelengths is a very fine measurement and hence the ability of the Michelson interferometer is aptly tested in their measurement.

Spectral Gap between Sodium Lines Method

A sodium lamp was placed at the input port of the interferometer with an additional pinhole tube placed afore the lamp. A telescopic tube was focused on an object at a large distance to ensure it was effectively focused at infinity and as such any incoming collimated rays converged to the telescope focal point (Hyperphysics 2018). Positioning the telescope at the output port the positioning of the pinhole tube was altered until in focus as viewed through the telescope. This ensured the rays entering the apparatus through the pinhole were accurately collimated. The telescope was then replaced by a detector array connected to an oscilloscope, able to detect the absence and presence of light upon its surface.

The oscilloscope was then calibrated to ensure a single full cycle of detection was clearly visible on the display screen. Vertical arm length was then adjusted via the micrometre head to place the output port at a point of minimum visibility, determined by the absence of peaks on the oscilloscope display. Visibility at the output port was then calculated for varying adjustments of the micrometre head where visibility is quantified by the expression (Hill, G 2018):

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Here I_{max} and I_{min} represent the values of maximum and minimum intensity detected by the detector array. To ensure a sufficient number of datapoints were collected, adjustments were made in increments of 0.1mm. This process was iterated until an ensuing point of minimum visibility was reached and the resulting data was then plotted using the Excel software

Results for Sodium Lines

Having measured the change in vertical arm length required to move from one point of minimum visibility to another the relation:

$$\Delta\lambda = \frac{\lambda^2}{2(D_2 - D_1)}$$

Can be used to find the difference between the two wavelengths ($\Delta\lambda$) of emission for sodium light. Here λ^2 is the mean wavelength of the two adjacent emission wavelengths and $D_2 - D_1$ is the distance between one point of minimum visibility to another. A derivation of this relation is included in the practical notes (Hill, G 2018). Following measurement the data recorded was displayed graphically using the Excel software as can be seen in diagram 2.

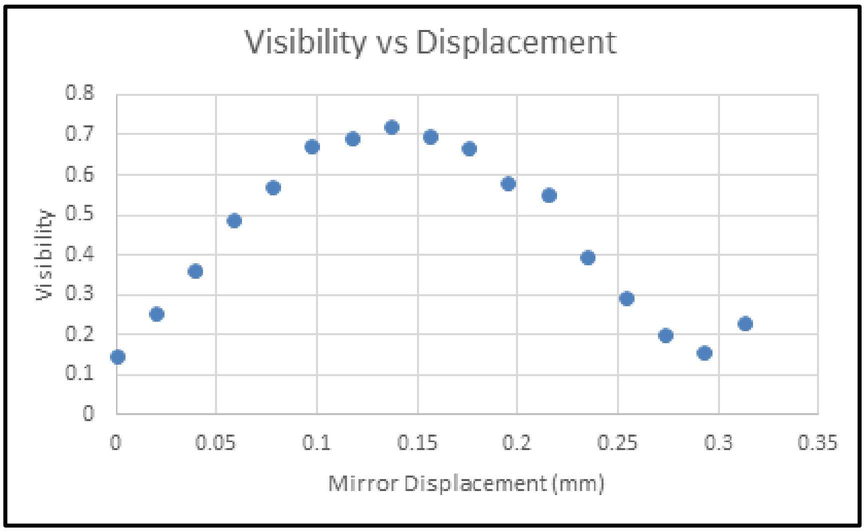

Diagram 2: Visibility vs Displacement of Vertical Arm for Sodium Lamp

It was thus observed the change in distance needed to move from one point of minimum visibility to another was:

$$|D_2 - D_1| = 0.280mm \pm 0.0025$$

Where uncertainty is given by half the smallest micrometer reading. From this the distance between adjacent Sodium wavelengths can be calculated to be:

$$\Delta\lambda = 0.62nm \pm 0.01$$

Here uncertainty is fundamentally limited by the minimum micrometre reading possible. For a complete selection of recorded data, uncertainty derivations and intermediate calculations see appendix section 2:A.

Furthermore, using the relation:

$$\Delta E = -h\frac{c}{\lambda^2}\Delta\lambda$$

The change in energy required to progress from one frequency to the next can be calculated where c is the speed of light and h is Planck's constant. For a full derivation of this relation see appendix section 2:B. And so:

$$\Delta E = \frac{1240eVnm}{(589.3nm)^2}(0.62nm) = 2.22meV \pm 0.04meV$$

Here uncertainty is limited by the uncertainty in $\Delta\lambda$ and a derivation of the exact value can be accessed via appendix section 2:A.

Discussion of results for Na Lamp

Due to the small nature of the wavelengths of the sodium emission spectrum of interest accurate determination of their values would prove strenuous were it not for the precision provided by the interferometer. In using the separation in wavelength to find the separation in energies the concluded results provide a remarkable insight into the inner workings of sodium's atomic structure. The two energies creating the energy difference arise from the quantised nature of atomic orbitals (Hyperphyscis 2018) and the fact that electron transitions between orbitals are accompanied by the release of photons at specific wavelengths, giving rise to the observed spectral lines.

Due to the precise nature of the Michelson interferometer uncertainty was mitigated to a sufficient enough level that nanometre distances and milli-electron volt energies were accurately determined. The only limiting variable in this regard was the increments discernable by the micrometre wheel, in this case 0.0025mm. To further decrease the uncertainty in the measured values a finer incremental device must be used however for the purposes of this investigation the gathered results prove highly satisfying.

Spectral Distribution of LED

Due to the polychromatic nature of light emitted via the chosen LED, the wavelengths produced are observed spread across a number of values as opposed to being primarily focused at a single wavelength, as seen previously (Markings, S 2018). Due to the fine length scale of the wavelengths produced measurement of the spread of the distribution requires precise instrumentation and hence the Michelson interferometer is ideally suited for this task.

Spectral Distribution of LED Method

Following the previous measurement of sodium spectral lines the equipment was re-calibrated to ensure optimum setup is once again reached. As explained prior this included focusing the telescope at infinity and then adjusting the pinhole tube such that when viewed through the telescope the pinhole was in focus. The vertical arm length was then adjusted until the fringe pattern observed on the oscilloscope was clearly visible. The mirror M_2 was then adjusted via rotation about the z-axis (see figure 1 for coordinate definitions) such that interference produced 6 distinct peaks on the oscilloscope display. The apparatus is now calibrated and values of visibility can be measured against adjustments of the vertical arm length.

To gain an estimate of the width of the spectral distribution the distance between adjacent points of minimum visibility was estimated to provide insight into the interval size which should be measured per increment. Resulting from this increments of 0.01mm were used for measurement giving a total of 15 data points between the adjacent minima. For every adjustment of the micrometre head the maximum and minimum voltages for an adjacent peak and trough on the oscilloscope screen were recorded. This measurement could then be used to find a value for visibility at that arrangement. Once all 15 data points were collected the results were graphed in the excel software and the value of full-width-half-maxima (FWHM) was calculated, given by the displacement of one half maxima point to another. Provided no point existed at the half maxima mark the intercept of the half maxima level and line of best fit was used.

The value for full-width-half-maxima could then be converted into a spectral distribution via use of the formula:

$$\Delta\lambda_{FWHM} = \frac{\lambda^2}{2\Delta D_{FWHM}}$$

Where the measured width of the distribution calculated prior is given by ΔD_{FWHM}. To calculate the average wavelength of the distribution, λ, the relation between fringe passes and micrometre adjustment:

$$\lambda = \frac{2|R_2 - R_1|}{m}K$$

Was utilised. The measurement of micrometre adjustment needed for the passage of 5 fringes was recorded and used to find a value for the average λ of the distribution.

Spectral Distribution of LED Results

Upon estimation it was discovered the micrometre adjustment needed to move between points of consecutive minimum visibility was 0.12mm and as such to ensure at least 12 data points could be collected incremental intervals of 0.01mm were chosen for adjustment. Once plotted in excel the data produced a bell shaped curve as seen in diagram 3.

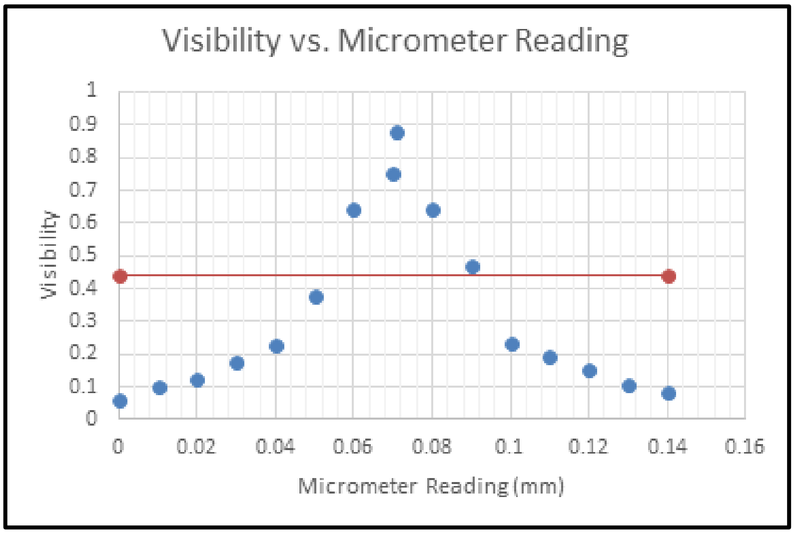

Diagram 3: Visibility vs. Micrometre Reading for LED

For the entirety of numerical data see appendix section 3:A. In order to add clarity to the data and provide an accurate value for the adjustment at half maximum, the vertical arm was set to a point of maximum visibility to add an additional data point at this value to the graph, as can be seen in diagram 3. Maximum visibility was observed to occur at a value of approximately 0.7mm with a corresponding visibility of 0.88. As such half maximum visibility is taken to be 0.44. The intercept between the best fit line constructed between point above and below the half maximum mark and the half maximum line itself were then measured and found to be:

$$Displacement\ at\ rising\ edge:\ D_1 = 0.052mm$$

$$Displacement\ at\ falling\ edge:\ D_2 = 0.091mm$$

FWHM was then calculated as:

$$\Delta D_{FWHM} = D_2 - D_1 = 0.091 - 0.052 = 0.039mm$$

In measuring the shift in micrometre position (R2 - R1) required for the passage of 6 fringes and averaging a value for the average wavelength of the light was calculated to be:

$$\lambda = \frac{2|R_2 - R_1|}{m} K = \frac{2 \times 0.009529}{6} (0.195)$$

$$= 619.4nm$$

From this the spread in spectral wavelengths could be calculated thus:

$$\Delta\lambda_{FWHM} = \frac{\lambda^2}{2\Delta D_{FWHM}} = \frac{(619.4)^2}{2 \times (7628.4)}$$

$$= 25.147nm$$

For a collection of raw data see appendix section 3:A.

Spectral Distribution of LED Discussion

Due to the small scale nature of spectral features accuracy in finding points of maximum and minimum visibility was hard to achieve however in the use of averaging and using best fit lines, discrepancies arising from this fact were minimised to a level that did not significantly impact the final results. The limiting factor herein lay with the ability to judge visibility extrema on the oscilloscope display. Should further reduction of uncertainty be desired more precise oscilloscope readings would be required however for the purposes of this investigation the method utilised provided precision sound enough to distinguish the required spectral features.

Due to the light emitted by the LED consisting of many frequencies, interference is limited by a greater level than seen previously. Provided the OPD between both interferometer arms is approximately zero then all frequencies arrive in phase and produce an interference pattern. For small deviations of OPD however, the changed path length places all frequencies at a slightly different point in their oscillatory motion upon reaching the output port. This potentially results in de-coherence and limits the ability to discern an interference pattern resulting in a visibility close to zero. In contrast to previous examples this placed a limit on the region for which measurements could be taken and this fact was taken careful note of when setting up the apparatus.

Conclusion

Through the use of a Michelson Interferometer the spectral features of monochromatic Mercury, Dichromatic Sodium and Polychromatic LED light sources were analysed and understood in great detail. Using the interference patterns produced by light travelling through a Michelson interferometer lengths scales otherwise inaccessible were able to be examined, allowing for further analyse of atomic spectral features which would be hard distinguished otherwise. The extent to which these features could be examined and the understanding it yields lay strong testimony to the experimental prowess of the Michelson interferometer.

References

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Appendix

Section 1 – Optimisation of Apparatus Setup

1:A: Derivation of Lever Arm to OPD Proportionality

By adjusting the micrometre head by an amount $\Delta R = |R_2 - R_1|$ the length of the vertical arm is altered by this same amount relative to the horizontal arm. Knowing the condition for constructive interference to be:

$$m\lambda = \Delta l$$

Where m is the number of interference points passed, λ the wavelength of light and Δl is the distance passed. In our case the distance passes is proportional to the motion of the micrometer and hence:

$$\Delta l = K\Delta R = K|R_2 - R_1|$$

Where K is a constant of proportionality. It then follows:

$$m\lambda = K|R_2 - R_1|$$

1:B: Measured and Calculated Data for Optimisation of Apparatus

Determination of Lever Ratio, k						
R_1 (mm)	R_1 Uncertainty (mm)	R_2 (mm)	R_2 Uncertainty (mm)	ΔR (mm)	m	k
9.0000	0.00025	8.9860	0.00025	0.0140	10	0.1950
8.7277	0.00025	8.7140	0.00025	0.0137	10	0.1993
8.7140	0.00025	8.6995	0.00025	0.0145	10	0.1883
8.7130	0.00025	8.7270	0.00025	0.0140	10	0.1950
8.7650	0.00025	8.7505	0.00025	0.0145	10	0.1883
8.5650	0.00025	8.5795	0.00025	0.0145	10	0.1883
8.5795	0.00025	8.5655	0.00025	0.0140	10	0.1950
8.5655	0.00025	8.5520	0.00025	0.0135	10	0.2023
8.5390	0.00025	8.5250	0.00025	0.0140	10	0.1950
8.5250	0.00025	8.5115	0.00025	0.0135	10	0.2023
8.7250	0.00025	8.6970	0.00025	0.0280	20	0.1950
8.6970	0.00025	8.6695	0.00025	0.0275	20	0.1986
8.6695	0.00025	8.6415	0.00025	0.0280	20	0.1950
8.6415	0.00025	8.6135	0.00025	0.0280	20	0.1950
8.6135	0.00025	8.5860	0.00025	0.0275	20	0.1986
8.5860	0.00025	8.5585	0.00025	0.0275	20	0.1986

Final Values and Uncertainties	
Uncertainty in k (mm)	8.594
λ (mm)	5.46E-04
Mean (mm)	0.02425
Standard Deviation	0.006486
Standard Error of the Mean	0.001622
Standard Deviation of the Mean	6.686855

Section 2 – Spectral Gap between Sodium Lines

2:A: Raw Data for Measurement of Sodium Wavelengths

Raw Data for Measurement of Sodium Wavelengths					
R_1 (mm)	ΔR (mm)	Mirror Displacement (mm)	Minimum Intensity (V)	Maximum Intensity (V)	Visibility
9.432	0	0	0.98	1.32	0.147826
9.53	0.098	0.019169	0.88	1.48	0.254237
9.63	0.198	0.038729	0.72	1.54	0.362832
9.73	0.298	0.058289	0.62	1.8	0.487603
9.83	0.398	0.077849	0.5	1.84	0.57265
9.93	0.498	0.097409	0.4	2.04	0.672131
10.03	0.598	0.116969	0.36	2	0.694915
10.13	0.698	0.136529	0.34	2.12	0.723577
10.23	0.798	0.156089	0.38	2.14	0.698413
10.33	0.898	0.175649	0.38	1.92	0.669565
10.43	0.998	0.195209	0.48	1.82	0.582609
10.53	1.098	0.214769	0.52	1.8	0.551724
10.63	1.198	0.234329	0.76	1.76	0.396825
10.73	1.298	0.253889	0.82	1.5	0.293103
10.83	1.398	0.273449	0.94	1.42	0.20339
10.93	1.498	0.293009	1.02	1.4	0.157025
11.03	1.598	0.312569	0.94	1.5	0.229508

Final Values and Uncertainties for Sodium Wavelengths	
$\Delta\lambda$ (μm)	0.62087538
Relative Uncertainty in $\Delta\lambda$	0.01951115
Uncertainty in $\Delta\lambda$ (μm)	0.012113993
Δf (Hz)	15375823645
Relative Uncertainty in Δf	0.01951115
Uncertainty in Δf (Hz)	9427568.175
ΔE (meV)	-0.00522031
Relative Uncertainty in ΔE	0.01951115
Uncertainty in ΔE (meV)	-4.33E-05
Mean (mm)	0.279665
Standard Deviation	0.021826
Standard Error of the Mean	0.005457
Relative Uncertainty in ΔD	0.019511

Determination of Sodium Wavelength Raw Data		
R_1 (mm)	ΔR(mm)	ΔD (mm)
13.842	1.016	0.19873
15.251	1.409	0.2756
13.7915	1.4595	0.285478
15.2315	1.44	0.281664
16.6935	1.462	0.285967
15.29	1.4035	0.274525
13.76	1.53	0.299268
15.256	1.496	0.292618
13.762	1.494	0.292226
15.2385	1.4765	0.288803
16.6895	1.451	0.283816
18.162	1.4725	0.288021
19.5935	1.4315	0.280001
21.0305	1.437	0.281077
22.5025	1.472	0.287923
23.9285	1.426	0.278926

2:B: Derivation of Energy Change Relationship

Given:

$$\frac{\Delta E}{\Delta \lambda} = \frac{dE}{d\lambda}$$

By taking derivatives and utilising the energy to wavelength relation we have:

$$\Delta E = \frac{dE}{d\lambda}\Delta\lambda$$

$$= \frac{d}{d\lambda}\left(\frac{hc}{\lambda}\right)\Delta\lambda = -\frac{hc}{\lambda^2}\Delta\lambda$$

Section 3 – Dispersion of LED

3:A: Raw Data for LED

Raw Data for LED Dispersion					
R_1 (mm)	ΔR (mm)	Mirror Displacement (mm)	Minimum Intensity (V)	Maximum Intensity (V)	Visibility
0.2	0	0	0.96	1.08	0.058824
0.21	0.01	0.001956	0.88	1.08	0.102041
0.22	0.02	0.003912	0.84	1.08	0.125
0.23	0.03	0.005868	0.84	1.2	0.176471
0.24	0.04	0.007824	0.76	1.2	0.22449
0.25	0.05	0.00978	0.6	1.32	0.375
0.26	0.06	0.011736	0.36	1.64	0.64
0.27	0.07	0.013692	0.24	1.68	0.75
0.28	0.08	0.015648	0.36	1.64	0.64
0.29	0.09	0.017604	0.48	1.32	0.466667
0.3	0.1	0.01956	0.72	1.16	0.234043
0.31	0.11	0.021516	0.76	1.12	0.191489
0.32	0.12	0.023472	0.8	1.08	0.148936
0.33	0.13	0.025428	0.84	1.04	0.106383
0.34	0.14	0.027384	0.88	1.04	0.083333
0.271	0.071	0.013888	0.12	1.88	0.88

Determining Average Wavelength Raw Data				
R_1 (mm)	R_2 (mm)	m	k	λ (nm)
8.7725	8.78	5	0.195609	586.8284
8.779	8.771	5	0.195609	625.9503
8.7705	8.7625	5	0.195609	625.9503
8.7635	8.756	5	0.195609	586.8284
8.755	8.763	5	0.195609	625.9503
8.763	8.7545	5	0.195609	665.0722

Intercept Values and Wavelength Spread for LED	
Intercept 1 (mm)	0.052
Intercept 2 (mm)	0.091
Micrometre Displacement (mm)	0.039
Mirror Displacement (nm)	7628.4
λ (nm)	619.4
λ_{FWHM} (nm)	25.15