On the Analysis of Disparate Circuit Filters

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Introduction

The modern world is built upon the principles of electronics and in many cases has direct roots to even the simplest circuits. Due to this, understanding the physical processes underpinning circuit function is an invaluable tool in developing and maintaining modern appliances and keeping their optimum functionality. Among these circuits Inductors, resistors and capacitors form a strong foundation for understanding and manipulating signals. Their use cannot be understated.

Single Loop AC Circuit Analysis

Kirchhoff's Laws

Kirchhoff's junction and loop laws govern the changes in voltage and current within a circuit and are an incredibly useful tool in circuit analysis. Kirchhoff's loop law (KLL) states that changes in voltage around any closed path within a circuit must sum to zero (Ottoway 2018) and is represented mathematically as:

$$\sum_{Closed\ Loop} V = 0$$

Kirchhoff's Junction Law (KJL) pertains to the behaviour of current and states that at any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction (Ottoway 2018). Again this can be expressed mathematically:

$$\sum I_{in} = \sum I_{out}$$

Given a basic circuit comprising only of resistors whose opposition to current flow is without time dependence, both laws hold (Hyperphysics 2018) however their application to more complicated time variant components, such as capacitors, is of interest and provides seed for the ensuing investigation.

Kirchhoff's Loop and Junction Law analysis method

To analyse the efficiency of application of Kirchhoff's Loop Law to time-variant components, in particular capacitors, an RCR circuit was constructed (Figure 1).

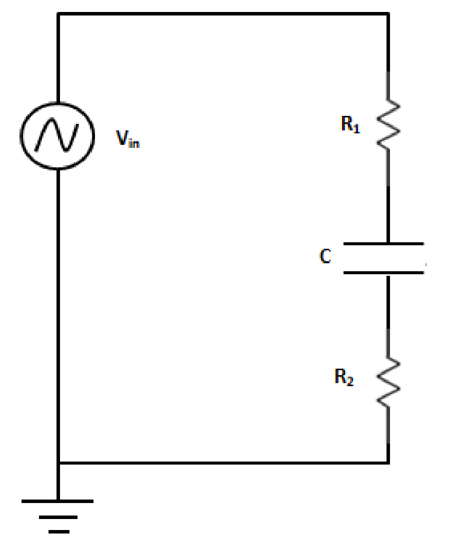

Figure 1: RCR Circuit Diagram

A signal generator was used to supply an alternating current of adjustable frequency to the circuit with an adjustable voltage setting. Resistor one (R_1) was chosen to have a resistance of approximately 1kΩ. To construct resistor two (R_2) two resistors (R_3 and R_4) of resistance equivalent in magnitude to resistor one were utilized in parallel. A capacitor was selected with a capacitance value of 100nF which was verified by the colour-code labelling upon the capacitor noting that for the purposes of this investigation precision in capacitance value is not crucial. Exact resistance values were measured using an LCR meter to be:

$$R_1 = 970.0\Omega \pm 0.5\Omega$$

$$R_2 = R_3 + R_4 = 970.0 + 970.0 = 1940.0\Omega \pm 0.7\Omega$$

Where uncertainties in resistance are taken as half the smallest increment measureable via the LCR meter. For derivation of uncertainty in the sum of resistances see Appendix section A:1.

Using a variety of signal generator frequencies ranging from 10Hz up to 100kHz with single order of magnitude spacing, voltage drops over each component were calculated by observing the peak-to-peak voltage given by the Oscilloscope display. Probes from the oscilloscope were connected either side of the signal generator to measure the voltage given by V_{in}, taking care the 0V probe was connected to ground. For resistor R_2 voltage drop was measured by attaching the oscilloscope probes to either side of the resistor. The two resistors were then interchanged to measure the voltage drop over resistor R_1 using the same process as that for the previous resistor. Returning the circuit to its original setup (Figure 1) the voltage drop was then calculated over the series component of R_2 and the capacitor C. To find the potential difference over the capacitor the voltage differences over the second resistor were then deducted from these measurements. The collected results were then compared with that predicted by KLL theory.

To asses KJL theory and its accuracy the currents passing through the resistors and capacitor for the aforementioned frequency range were calculated. This was done by utilising Ohms Law in impedance form and solving for current as follows:

$$V = IZ \qquad so \qquad I = \frac{V}{Z}$$

Where V represents voltage drop over the component, I the current through the component and Z the impedance of the component. For a resistor the impedance is simply given by its resistance. Due to the time dependence in voltage and current of a capacitor, caused by its charging, the impedance is given by:

$$Z_c = \frac{1}{i\omega C}$$

Where the complex unit i encapsulates the time dependence of the capacitors charging and hence its impedance. Here C is the capacitance of the capacitor and ω is the angular frequency of the current. Given the voltage drops per component for particular frequencies calculated prior Ohm's Law in impedance form could then be combined with the known impedances to find the current through each component. These currents were then compared with that predicted by KJL theory.

Results for Kirchhoff's Law Analysis

In calculating the sum of peak-to-peak voltages around the circuit for different frequencies it was observed KLL theory was matched for high and low frequencies as 0V sums were recorded within experimental error (see appendix section A:2 for tabulated data). For mid-range frequencies (100Hz-10,000Hz) however, results overshot theory by an extrema of 58.3±0.2V while theory predicts a constant result of 0V for all frequencies.

Comparing currents over each component it was found all currents were approximately equal and all were within the same order of magnitude for a given frequency range (See appendix section A:2 for raw data). This result matches that expected by theory as all components are connected in series and hence KJL stipulates all currents should be equal.

Discussion of Results for Kirchhoff's Law Analysis

Discrepancy between the measured results and theoretical prediction by KLL theory is minimized for high ($\gtrsim$ 100kHz) and low ($\lesssim$ 10Hz) frequencies but compounds for mid-range frequencies (10Hz – 10,000Hz). This is due to the measured peak-to-peak voltages representing the maximum magnitude of the voltages over each component per cycle but not considering their respective time or phase offsets which are known to be shifted by 90° by theory (Ottoway 2018). For lower frequencies theory is more accurately matched as the impedance of the capacitor becomes increasingly large and hence most voltage is dropped over the capacitor, which is easily seen by considering the mathematics of the capacitors impedance (see Appendix section A:3). Due to this the vector sum of both voltages is approximately equal to the magnitude of the voltage drop over the capacitor, and so phase difference need not be considered for accuracy in using KLL (Hyperphysics 2018). A similar case holds for high frequencies as capacitor impedance tends to infinity and so almost all voltage is dropped over the resistor, making the vector sum of both voltage drops approximately equal to that over the resistor.

KJL is observed to hold for the measured values of current throughout the circuit over all frequency ranges. At a given instant the current over the capacitor and resistors may be different due to the charging of the capacitor affecting its impedance and hence current by Ohms law. However, the peak-to-peak current calculated is only respective of the maximum current experienced per cycle by each component and as such the time difference in currents is not considered. This results in KJL theory being an accurate model for the peak-to-peak currents within the circuit.

High Pass Filter Analysis

Frequency filters are a widely useful circuit which allow for the manipulation and control of electrical signals within AC circuits. In a fundamental form the desired filtering effect can be achieved via the use of a single resistor and capacitor with the filtered signal being output using voltage divider principles. This setup in High Pass Filter form is displayed in figure 2.

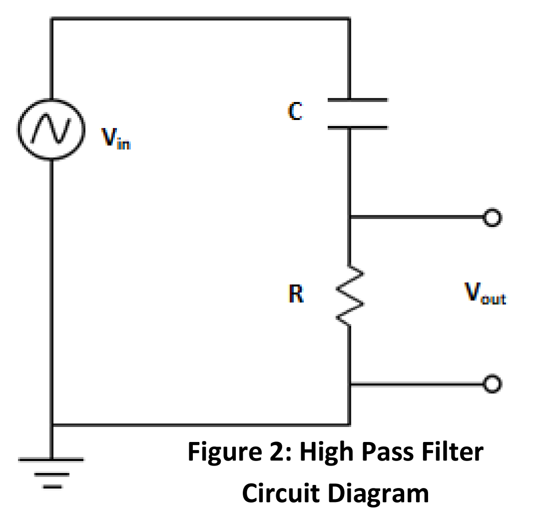

Figure 2: High Pass Filter Circuit Diagram

Theoretical functionality can be derived taking the output voltage to be given by the voltage divider equation (Ottoway 2018). Applying this logic to the High Pass Filter circuit we see the transfer function is given by:

$$\frac{V_{out}}{V_{in}} = \frac{Z_R}{Z_C + Z_R} = TF$$

Where Z_R represents the impedence of the resistor which is known to be simply its resistance R, and Z_C is the impedance of the capacitor which was derived prior (see Kirchhoff's analysis). Substituting these values into the voltage divider equation we arrive at a theoretical model for the transfer function as a function of angular frequency ω:

$$TF(\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{\frac{1}{i\omega C} + R} = \frac{i\omega CR}{1 + i\omega CR}$$

Where C is the capacitance of the capacitor and R the resistance of the resistor. The effect of a DC offset in the applied voltage and its effect on circuit functionality is also of interest for this circuit.

The extent to which the derived theoretical model represents reality and the physical principles underpinning any inaccuracy provide a basis for investigation.

High Pass Filter Analysis Method

A High Pass Filter Circuit is constructed for investigation (Figure 2) utilising a capacitor with capacitance of 980.4nF and resistor with resistance of 977.4Ω. Oscilloscope probes are attached to either side of the resistor component to function as the output voltage, taking care the 0V probe is connected to ground. Values of input voltage, given by the signal generator display, are then measured against output voltage for a variety of AC frequencies set by the signal generator. Due to the logarithmic nature of the frequency-voltage relationship frequencies were selected with a logarithmic scale in mind at values of 1 and 3 for every order of magnitude ranging from 10Hz to 100kHz. Phase difference between output and input voltage was measured by analysing the time difference between characteristic waveform points on the signal generator for the above mentioned frequencies. To minimise uncertainty in recorded time values the points of maximum gradient on the waveform were selected for time delay measurement. Observed values were then compared with that expected by theory via the MATLAB analytical software to give insight into any mismatch between theory and experiment.

A DC offset voltage of 1V was then added to V_{in} by setting the DC offset setting on the signal generator to be at the 1V marking. The resulting shift in output voltages was observed for the aforementioned frequencies to give insight into the effect of the DC offset.

Results for High Pass Filter Analysis

Low frequencies of 10Hz resulted in a TF value of approximately 0.06 with a gradual increase in amplitude from frequencies of 10Hz up to 3kHz. Following this a plateau in amplitude of 10.5V ± 0.2V was reached for frequencies thereafter (see Appendix section B:1) which exactly matches input voltage within experimental error and suggests a TF value of 1. These results matched that expected by the theoretical TF derived previously for both high and low frequency ranges since:

$$\lim_{\omega\to\infty} TF(\omega) = \lim_{\omega\to\infty} \frac{i\omega CR}{1+i\omega CR} = 1 \qquad and \qquad \lim_{\omega\to 0} TF(\omega) = \lim_{\omega\to 0} \frac{i\omega CR}{1+i\omega CR} = 0$$

Where low frequencies are represented by ω tending to 0 and high frequencies by ω tending to infinity. These results were further clarified by MATLAB graphs displaying the experimental measurements overlaid with theoretical curves (Figure 3).

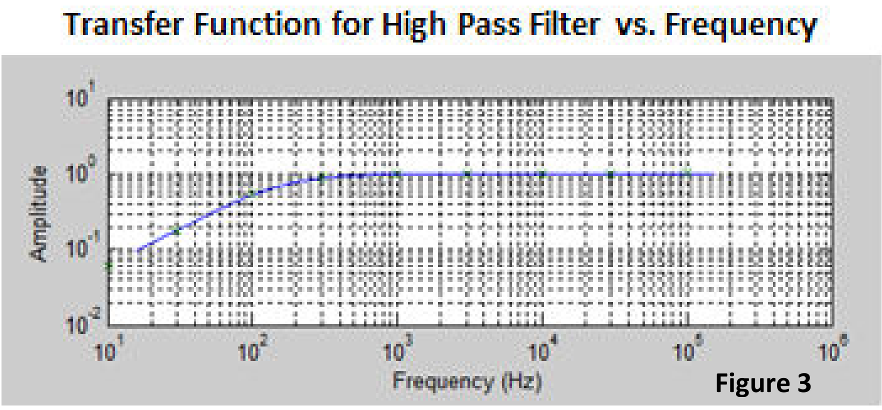

Figure 3

Phase offset is observed to be maximised (25ms ± 1ms) for lower frequencies of 10Hz with a continual decline to a minimum offset (0.00ms ± 0.01ms) at the 100kHz mark (these results are tabulated in detail in Appendix section B:1). This relationship is expressed explicitly in the constructed MATLAB plots (Figure 4).

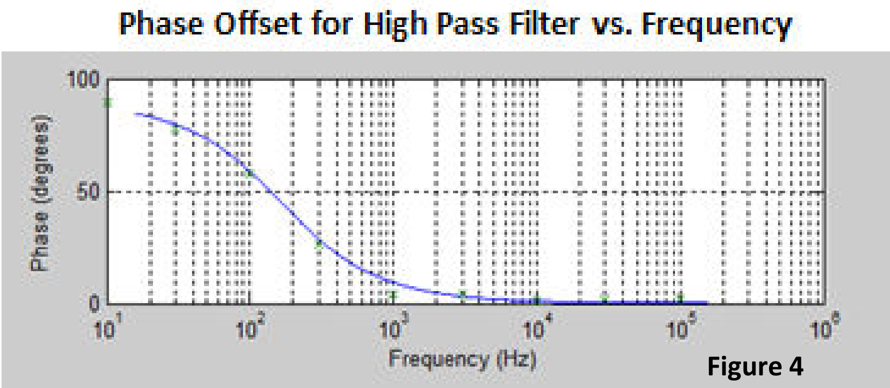

Figure 4

DC offset produces no shift in the recorded output voltages despite having a constant offset represented on the signal generator display. This pattern was observed to hold and produced the same trend as seen previously with a constant offset included for measurements of V_{out}.

Discussion of Results for High Pass Filter Analysis

At low frequencies the transfer function approaches zero and continues to climb as frequency increases until a constant value of approximately 1 is reached around the 1.5kHz mark. This can be linked physically to the build-up of charge on the capacitors plates. For significantly low frequencies the applied AC current begins to replicate a DC current. In this regime sufficient time elapses such that the charge held on the capacitor impairs current flow and the capacitor begins to function as an open circuit. Due to this the majority of applied voltage is dropped over the capacitor reducing the voltage dropped over the resistor and hence minimising the transfer function (seen mathematically in Appendix section A:3).

As frequencies climb the reverse process occurs as the fast alternating current prevents significant charge build up within the capacitor and hence the opposition to current flow provided by the capacitor is minimal. In turn little voltage is dissipated over the capacitor leaving almost all voltage dropped over the resistor leading to a maximised transfer function of 1.

The charging of the capacitor also acts to impose the observed phase shift between the input and output voltage. As the input voltage is maximised the rate of capacitor charging is maximised and the capacitor continues to charge for some time until it reaches maximum charge. Due to this time delay caused by the build-up of charge on the capacitor there is a corresponding phase offset between the amplitude of the input voltage and that measured at the output. As frequencies become increasingly large however significant time is not able to elapse for charge to accumulate on the capacitor, causing it to act as an open circuit and hence the phase offset between input and output voltage is minimised as a result.

A DC current is analogous to an AC current with a frequency of 0 meaning it does not repeat for all time. Adding a DC offset to the AC voltage then only acts to charge the capacitor by the amount of the offset, in this case 1V. As this additional voltage is not able to progress past the capacitor in the High Pass circuit once charged it has no effect on the output voltage, despite increasing the input voltage, as was observed experimentally.

Band Pass Filter Analysis

A useful circuit in the control and filtering of electrical current is the Band Pass Filter which utilises an inductor and capacitor in parallel to siphon out all but a specific range of frequencies. Theoretical understanding of such a circuits behaviour can be captured via the use of a transfer function which gives the relationship between the outputted filtered voltage and the input voltage. Again using voltage divider theory the transfer function for a Band Pass Filter can be shown to be:

$$TF(\omega) = \frac{1}{1 + iR\left(\omega C - \frac{1}{\omega L}\right)}$$

Where ω is the angular frequency of the current, R the resistors resistance, L the inductance of the inductor and C the capacitance of the capacitor (see appendix section C:1 for a full derivation). From this expression it can be observed there are 3 frequencies of interest. As frequencies tend to infinity and zero we observe the Transfer Function tending to zero:

$$\lim_{\omega \to 0} \frac{1}{1 + iR\left(\omega C - \frac{1}{\omega L}\right)} = \lim_{\omega \to \infty} \frac{1}{1 + iR\left(\omega C - \frac{1}{\omega L}\right)} = 0$$

It can also be seen that for:

$$\omega = \sqrt{\frac{1}{LC}}$$

The transfer function reaches its maximum value of 1 (See Appendix section C:2 for a proof). This frequency is known as the characteristic frequency of the circuit. The extent to which these theoretical predictions accurately represent the real world is of interest and gives insight into any corrections to theory that may be needed to better represent reality.

Band pass Filter Method

To analyse circuit behaviour a band pass filter was constructed with the form displayed in figure 5.

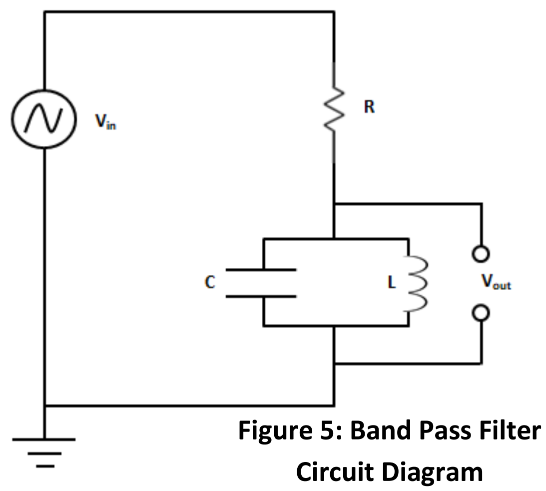

Figure 5: Band Pass Filter Circuit Diagram

A resistor of value 974.6±0.05Ω, a capacitor with capacitance 1.048±0.0005μF and inductor with inductance of approximately 100μH were utilised in the circuit (Inductance is here given approximately as the original inductance value was recorded as 732±0.5μH but upon analysis of results was observed to actually be approximately 100μH). All values were measured via an LCR meter. Oscilloscope probes were then connected to either side of the parallel circuit component to provide a measure of V_{out}, taking care the 0V probe was connected to ground. Measurements of V_{out} were then taken and compared with measurements of V_{in} (given by the signal generator display) for frequencies ranging from 10Hz up to 100kKz. Values of 3, 5 and 10 were selected per order of magnitude of frequency with additional values chosen around the characteristic frequency f_c given by:

$$\omega_c = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(100 \times 10^{-6})(1.048 \times 10^{-6})}} = 97.68 \times 10^3 s^{-1}$$

$$So: \quad f_c = \frac{\omega_c}{2\pi} = \frac{97.68 \times 10^3}{2\pi} = 15.55 kHz$$

This frequency allowed for circuit filter behaviour to be optimally captured.

The resistor was then replaced to have a resistance value of approximately 500Ω and the above mentioned process was repeated to determine the transfer function across a wide (10Hz up to 100kHz) frequency range. Experimental values were then graphically overlaid with the theoretical curve using the MATLAB analytical software to give insight into any discrepancies between the two.

Results for Band Pass Filter Analysis

Transfer Function was minimised for both high and low frequencies but peaked around the characteristic frequency range, as is tabulated in the recorded data (see Appendix section C:1). When overlaid with the theoretical curve this relationship is further evident as seen by the produced graph (Figure 6).

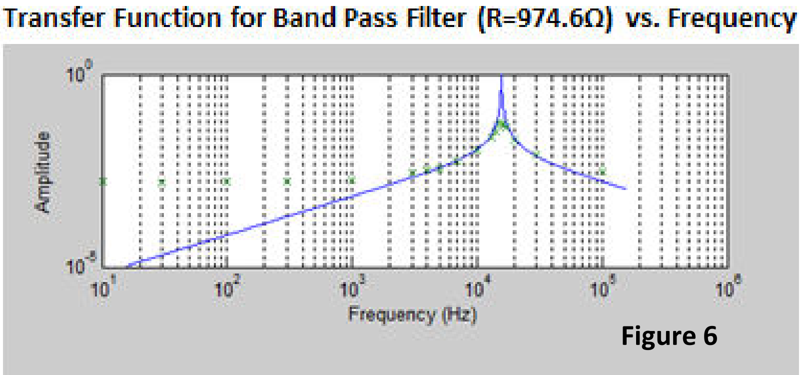

Figure 6

Upon interchanging the resistor with a resistor of resistance 500Ω a similar pattern is observed with high and low frequencies giving a minimum transfer function and frequencies close to the characteristic frequency giving maximum TF of approximately 1. This can be seen from the tabulated data (Appendix section C:1) and the graph constructed using the MATLAB analytical software (Figure 7).

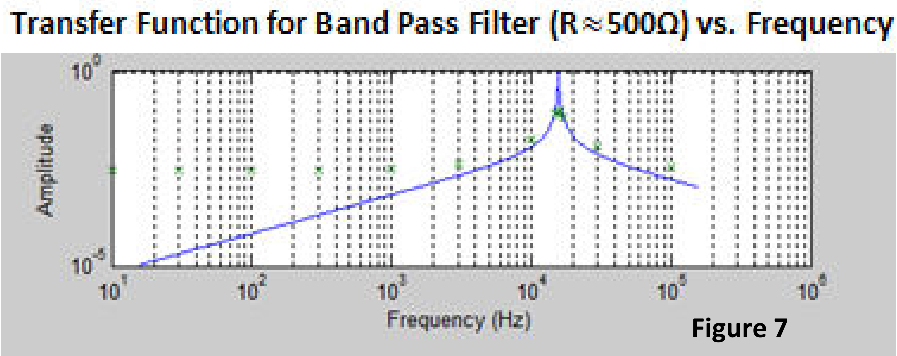

Figure 7

In both cases an interesting mismatch between theory and experiment is observed as frequencies tend toward zero. By adding a constant offset in the value of the inductor of 1.54 this plateau was corrected. The results then produced the graph displayed in figure 8.

Mathematically the transfer function is then observed to be of the form:

$$TF(\omega) = \frac{i\omega L + Q}{i\omega L + Q + R(-\omega^2 CL + i\omega CQ + 1)}$$

Where Q represents the constant offset for inductance (See appendix section C:3 for a full derivation).

Transfer Function for Band Pass Filter (R≈500Ω) with adapted Inductance vs. Frequency

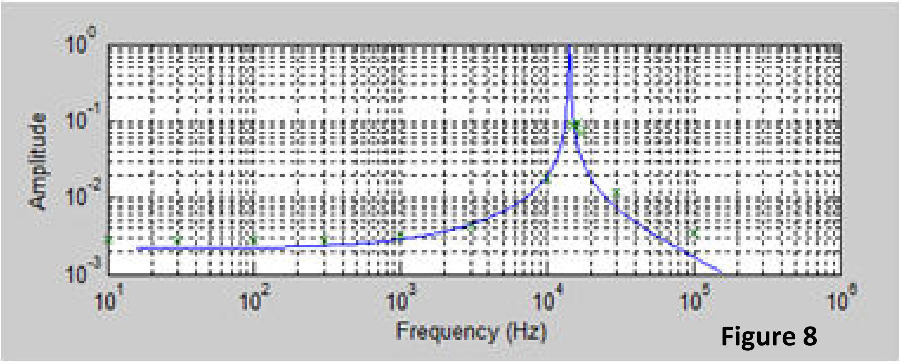

Figure 8

Discussion of Results for Band Pass Filter Analysis

Comparing theory and experiment correlation was observed for frequencies above the 1.5kHz mark for circuits with both resistors of 500Ω and 974.6Ω. Here a peak at the characteristic frequency of 15.55kHz was seen giving a TF value of 1 followed by a gradual decrease thereafter for frequencies up to 100kHz. Below this frequency range discrepancies compound as theory suggests a tail towards a TF of zero whereas experiment had a TF plateau at just above the 10^{-3} amplitude mark. This difference was due to the inductor being non-ideal and contributing an internal resistance which acted to impair current flow and prevent the 0 amplitude mark being reached. Noting that this altercation was caused by resistance within the inductor the theoretical transfer function can then be edited to reflect this now using R_L as the resistance of the inductor:

$$TF(\omega) = \frac{i\omega L + R_L}{i\omega L + R_L + R(-\omega^2 CL + i\omega C R_L + 1)}$$

Transient Response of a Band Pass Filter

The response of circuits to sudden step changes in voltage is another circuit trait of interest which provides insight into the physical inner workings of circuit components. As such its investigation is useful to understanding the physics and functionality of particular circuits. The specific case of a band pass filter will be analysed here to provide further insight into how such filters operate and gain a better understanding of the principles governing them.

Transient Response of a Band Pass Filter Method

A band pass filter, seen in figure 5, was constructed for further analysis. An inductor with inductance 101.9μH (±0.1μH), capacitor with capacitance 95.8nF (±0.1nF) and resistor with resistance 978Ω (±0.5 Ω) were utilized. In the analysis of the Band Pass Filter the filtering nature of the circuit is most prominent at the characteristic frequency (Ottoway 2018). For the constructed circuit this was given by:

$$\omega_c = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(101.9 \times 10^{-6})(0.958 \times 10^{-6})}} = 101.212 \times 10^3 s^{-1}$$

$$So: \quad f_c = \frac{\omega_c}{2\pi} = \frac{101.212 \times 10^3 s^{-1}}{2\pi} = 16.11 kHz$$

To ensure the transient nature of the circuit is captured a circuit frequency significantly lower than the characteristic frequency (5 kHz) was chosen to enable the behaviour to be easily observable within one current cycle.

Oscilloscope probes were then attached either side of the parallel circuit component to measure V_{out} as in the original analysis of the band pass filter again taking care the 0V probe was connected to ground. Input voltage frequency was set to 5kHz via the signal generator settings and the circuit response given by the oscilloscope display was observed and plotted.

Transient Response Results

Oscilloscope display matches that expected for an exponentially damped oscillatory system and is shown in figure 9. This represents one cycle of behaviour, after which the same behaviour is observed in reverse, with amplitude first decreasing to a minima and then exponentially decaying whilst oscillating around the zero point.

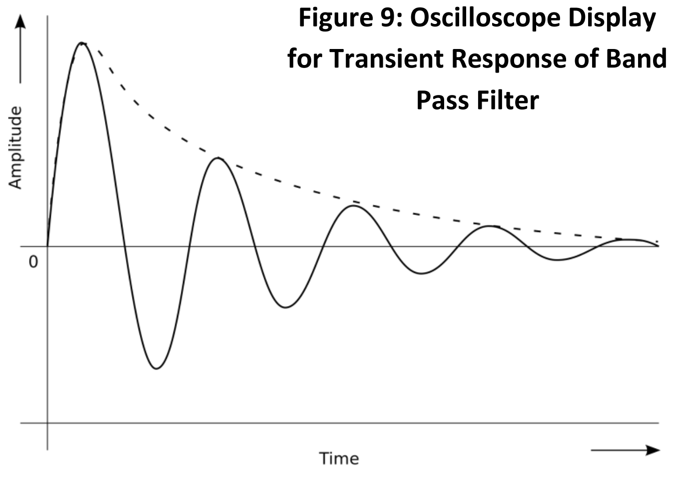

Figure 9: Oscilloscope Display for Transient Response of Band Pass Filter

Five maxima are reached in the transfer function amplitude before the signal completely decays and then repeats. The total time for one cycle is observed to be 100μs.

Transient Response Discussion

Transient response analysis reveals a second order ODE with exponentially damped sine and cosine solutions for the voltage of the output which matches the response seen in the oscilloscope display. As time progresses within one cycle the applied current acts as a DC source. Due to this the capacitor becomes fully charged, acting then as an open circuit while the original resistance of the inductor is overcome as a B field is constructed within its coils and the inductor functions as a short circuit. This behaviour is quantified by analysing the impedance formulas for each component as time tends to infinity (see Appendix sections A:3 and D:1).

Altering the direction of current flow at the end of each cycle causes the charge stored within the capacitor to be dissipated back toward the signal generator with the inductor acting as an open circuit due to the B field held within its coils providing a strong opposition to current flow. This behaviour of both components is further quantified by considering the effective resistance for short time scales (see Appendix sections A:3 and D:1).

Once the capacitor has discharged the inductor can then promote current flow in the opposite direction, hence the decrease in the waveform seen in figure 9, by virtue of the B field retained within its coils from the previous cycle. This current flow causes charge to once again build up on the capacitor which can then be dissipated causing the amplitude to flip once more. This process continues until the B field stored within the inductor is fully dissipated or the signal generator voltage inverts, restarting the cycle

Observing the transfer function of the circuit derived in the previous section we note that output voltage is maximised for the characteristic frequency of $\omega = (LC)^{-1/2}$ while divergence from this value causes a decrease in the outputted voltage over the LC parallel component.

Physically, the frequency dependant terms in the transfer function correspond to the build-up of charge on the capacitor and the storage of energy in the B field within the inductor.

Conclusion

Through the analysis of inductors, capacitors and resistors within multiple circuits it was seen current could be manipulated to exclude all but high frequencies and also allow a narrow bandwidth of frequencies within a specific range. This behaviour was found to be caused by the build-up of charge within the capacitor components and the storage of energy within the B fields setup within the inductor. Understanding of the physical principles underpinning the behaviour of components within these circuits provides the ability to better manipulate their functionality and correct any sub-optimal characteristics. As a result this knowledge is of much practical significance.

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Appendix

Section A

1: Derivation of Uncertainty Sum

Given the general formula for uncertainty calculations (Norton 2010) uncertainties can be found thus:

$$\sigma_{R_2}{}^2 = \sigma_{R_3}{}^2 + \sigma_{R_4}{}^2$$

$$\sigma_{R_2} = \sqrt{0.5^2 + 0.5^2} = 0.7$$

2: Tableted Data for Analysis of Kirchhoff's Laws

Frequency vs. Sum of Peak-to-peak Voltages Over All Components	
Frequency (Hz)	Voltage (V)
10	-1±1
100	-14.4±0.7
1,000	-58.3±0.2
10,000	-4±1
100,000	-5.8±0.7

Frequency vs. Peak-to-peak Voltage for Resistor (R_2)	
Frequency (Hz)	Voltage (V)
10	1.17±0.01
100	10.9±0.01
1,000	55.2±0.01
10,000	64.6±0.2
100,000	64.4±0.4

Frequency vs. Peak-to-peak Voltage for Resistor (R_1)	
Frequency (Hz)	Voltage (V)
10	0.6±0.8
100	5.48±0.04
1,000	28.2±0.2
10,000	33.8±0.2
100,000	33.6±0.4

Frequency vs. Peak-to-peak Voltage for Capacitor (C)	
Frequency (Hz)	Voltage (V)
10	100.0±0.1
100	98.00±0.05
1,000	76.0±0.1
10,000	66.0±0.1
100,000	66.0±0.1

Frequency vs. Peak-to-peak Voltage for Signal Generator (V_{in})	
Frequency (Hz)	Voltage (V)
10	100.0±0.5
100	100.0±0.5
1,000	98.40±0.05
10,000	101±1
100,000	98.8±0.4

3: Proof of Capacitor Voltage Response to Extreme Frequency Ranges

$$\lim_{\omega \to 0} Z_c = \lim_{\omega \to 0} \frac{1}{i\omega C} \to \infty \quad so \quad V_c = 0$$

$$\lim_{\omega \to \infty} Z_c = \lim_{\omega \to \infty} \frac{1}{i\omega C} = 0 \quad so \quad V_c \longrightarrow \infty$$

Section B

1: Tableted Data for Analysis of High Pass Filter

Oscilloscope Readings for High Pass Filter			
Frequency	V_{in} (V)	V_{out} (V)	Time Offset (ms)
10Hz	10.4±0.05	0.634±0.004	25±1
30Hz	10.4±0.05	1.83±0.01	7±1
100Hz	10.2±0.2	5.5±0.1	1.6±0.1
300Hz	10.0±0.2	8.9±0.1	0.24±0.05
1kHz	10.6±0.2	10.3±0.1	0.01±0.01
3kHz	10.7±0.1	10.5±0.1	0.00±0.01
10kHz	10.7±0.1	10.7±0.1	0.00±0.01
30kHz	10.7±0.1	10.6±0.2	0.00±0.01
100kHz	10.4±0.4	10.6±0.2	0.00±0.01

Section C

1: Transfer Function Derivation for Band Pass Filter

Voltage divider theory gives:

$$TF(\omega) = \frac{Z_{C||L}}{Z_{C||L} + Z_R}$$

Where parallel component impedance (represented by $Z_{C||L}$) is given by the parallel resistance sum law:

$$Z_{C||L} = \left(\frac{1}{Z_C} + \frac{1}{Z_L}\right)^{-1} = \left(\frac{1}{(i\omega C)^{-1}} + \frac{1}{i\omega L}\right)^{-1}$$

$$Z_{C||L} = \left(i\omega C + \frac{1}{i\omega L}\right)^{-1} = \frac{1}{i\omega C + \frac{1}{i\omega L}}$$

$$so \quad Z_{C||L} = \frac{i\omega L}{1 - \omega^2 LC}$$

Transfer Function is then:

$$TF(\omega) = \frac{\frac{i\omega L}{1-\omega^2 LC}}{\frac{i\omega L}{1-\omega^2 LC} + R} = \frac{1}{1 + \frac{R - \omega^2 RLC}{i\omega L}} = \frac{1}{1 + iR\left(\omega C - \frac{1}{\omega L}\right)}$$

2: Proof of Characteristic Frequency for Band Pass Filter

Given the characteristic frequency:

$$\omega = \sqrt{\frac{1}{LC}}$$

Transfer Function then becomes:

$$TF(\omega) = \frac{1}{1 + \frac{R - \omega^2 RLC}{i\omega L}} = \frac{1}{1 + \frac{R - (LC)^{-1}RLC}{i(LC)^{-1/2}L}} = \frac{1}{1 + \frac{R - R}{i(LC)^{-1/2}L}} = \frac{1}{1+0}$$

Therefore:

$$TF = 1$$

3: Derivation of Adapted Transfer Function Incorporating Inductor Resistance

Transfer function given by voltage divider to be:

$$TF(\omega) = \frac{Z_{CLR}}{Z_{CLR} + Z_R}$$

Where parallel component resistance is:

$$Z_{CLR} = \left(\frac{1}{Z_C} + \frac{1}{Z_{LR}}\right)^{-1}$$

$$Z_{CLR} = \left(i\omega C + \frac{1}{i\omega L + R_L}\right)^{-1} = \frac{i\omega L + R_L}{-\omega^2 CL + i\omega CR_L + 1}$$

So:

$$TF(\omega) = \frac{\frac{i\omega L + R_L}{-\omega^2 CL + i\omega CR_L + 1}}{\frac{i\omega L + R_L}{-\omega^2 CL + i\omega CR_L + 1} + R} = \frac{1}{1 + R\frac{-\omega^2 CL + i\omega CR_L + 1}{i\omega L + R_L}}$$

$$= \frac{i\omega L + R_L}{i\omega L + R_L + R(-\omega^2 CL + i\omega CR_L + 1)}$$

4: Tabulated Data for Analysis of Band Pass Filter

Analysis of Band Pass Filter (R=974.6Ω) Data		
Frequency	V_{in} (V)	V_{out} (V)
10Hz	9.2±0.08	0.016±0.0006
30Hz	9.2±0.08	0.016±0.0004
100Hz	9.2±0.05	0.016±0.0002
300Hz	9.4±0.05	0.016±0.0004
1kHz	9.2±0.05	0.017±0.0004
3kHz	9.2±0.05	0.03±0.01
4kHz	9.2±0.06	0.030±0.006
5kHz	9.6±0.005	0.0354±0.0002
7kHz	9.6±0.005	0.053±0.0008
10kHz	9.6±0.005	0.1±0.0002
13kHz	9.5+0.1	0.226±0.0002
14kHz	9.6±0.005	0.320±0.008
15kHz	9.7±0.1	0.468±0.004
15.3kHz	9.6±0.005	0.504±0.004
15.5kHz	9.6±0.005	0.502±0.004
16kHz	9.6±0.005	0.508±0.004
17kHz	9.6±0.005	0.388±0.004
20kHz	9.6±0.005	0.188±0.004
30kHz	9.4±0.2	0.076±0.004
100kHz	9.4±0.2	0.028±0.004

Analysis of Band Pass Filter (R=500Ω) Data		
Frequency	V_{in} (V)	V_{out} (V)
10Hz	9±0.005	0.025±0.0008
30Hz	9±0.005	0.0252±0.0008
100Hz	9±0.005	0.025±0.0004
300Hz	9±0.005	0.0256±0.0004
1kHz	9±0.005	0.0276±0.0002
3kHz	9.4±0.005	0.04±0.005
10kHz	9.4±0.2	0.168±0.004
15kHz	9.4±0.005	0.784±0.004
15.3kHz	9.4±0.005	0.832±0.004
15.5kHz	9.4±0.005	0.852±0.004
16kHz	9.4±0.005	0.888±0.006
17kHz	9.4±0.005	0.648±0.004
30kHz	9.4±0.005	0.112±0.001
100kHz	9.4±0.005	0.032±0.008

Section D

1: Proof of Inductor Voltage Response to Extreme Frequency Ranges

$$\lim_{\omega \longrightarrow 0} Z_L = \lim_{\omega \longrightarrow 0} i\omega L = 0 \quad so \quad V_L \longrightarrow \infty$$

$$\lim_{\omega \longrightarrow \infty} Z_L = \lim_{\omega \longrightarrow \infty} i\omega L \rightarrow \infty \quad so \quad V_L = 0$$