Particle Physics
Relativistic Quantum Mechanics and Particle Physics Assignment II

Joseph Pritchard - Adelaide University

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Question I - Feynman Diagrams

Note that within Feynman diagrams anti-quarks are labelled via a primed tag (u') as oppose to the usual horizontal bar ($\bar{u}$).

a) Reaction is permitted and occurs via the strong interaction, mediated by gluons. Feynman diagram is displayed in figure 1.

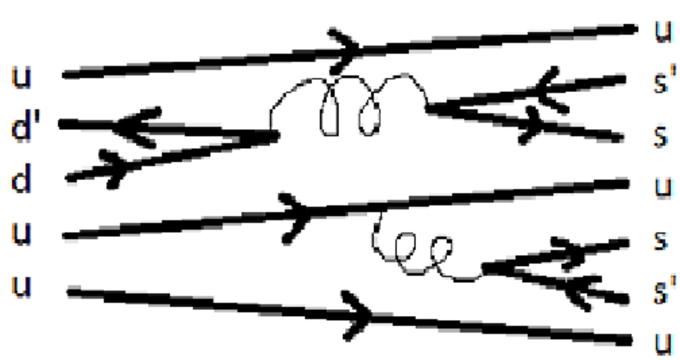

Figure 1:
Reactants include a proton (uud) and π^+ Meson ($u\bar{d}$). Products include two K^+ Mesons ($u\bar{s}$) and a Ξ^0 Baryon (sus)

b) Reaction is permitted and occurs via the weak interaction, where a strange quark is converted to a u,c or t quark following the emission of a W^- Boson. The Boson is reabsorbed converting the u,c or t quark into a down quark. While permeating, the W^- Boson emits a photon via the electromagnetic interaction. Feynman diagram is displayed in figure 2.

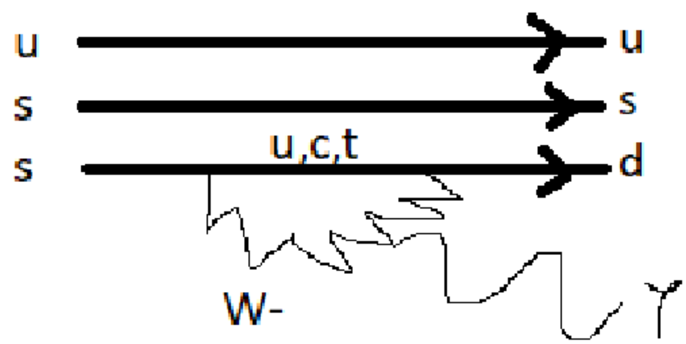

Figure 2:
Reactant is a Ξ^0 (uss). Product is Σ^0 (usd).

c) Reaction is not permitted by the standard model as neutrinos and anti-neutrinos have no charge so cannot couple to the electromagnetic field, preventing their creation from photons.

The reaction is however possible via an indirect reaction pathway involving the weak interaction

$$\gamma + \gamma \rightarrow e^+ + e^- \rightarrow Z \rightarrow \nu_e + \bar{\nu_e}$$

where the second incident photon is present to conserve momentum in the production of the electron positron pair. Feynmand diagram is displayed in figure 3.

d) Reaction is permitted and occurs via the strong interaction, mediated by gluons. Feynman diagram is displayed in figure 4.

e) Reaction is permitted and occurs via the weak interaction, mediated by the Z Boson. Two Z Bosons are responsible for the creation of the Higgs Boson H. Feynman diagram is displayed in figure 5.

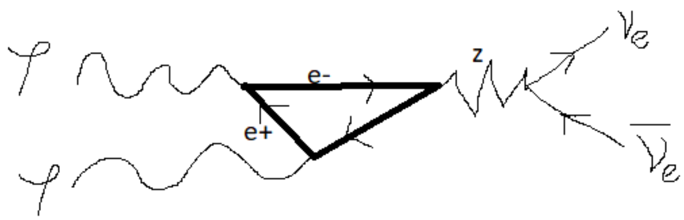

Figure 3:
Indirect production of neutrion antineutrino pair from two incident photons.

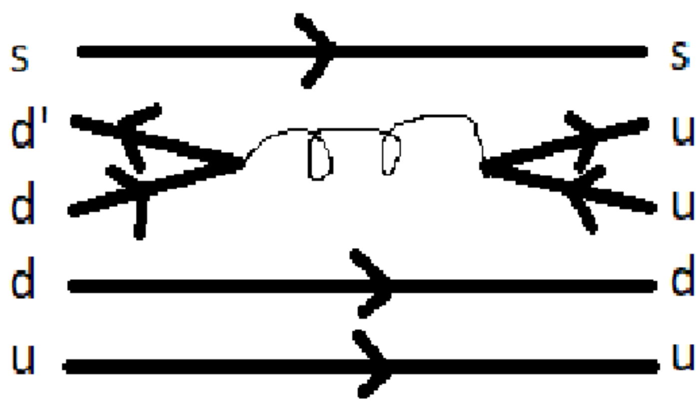

Figure 4:
Reactants are a K^0 Meson ($s\bar{d}$) and a neutron (ddu). Products are a K^- Meson ($s\bar{u}$) and a proton (udu).

Question II - SU(3) flavour and colour symmetry

a) i) By considering Isospin (I_3) and Hypercharge (Y) defined as:

$$I_3 = \frac{1}{2}(U + D)$$
$$Y = 2(Q - I_3)$$

where U and D are the up and down quantum numbers and Q is the charge, we may represent Mesons and Baryons in terms of these values and arrive at

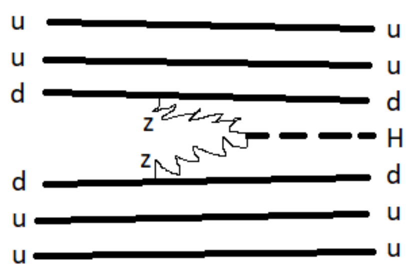

Figure 5: Reactans are two protons (uud). Products are two protons (uud) and the Higgs Boson H.

symmetrical representations. This representation is justified provided we assume low system energy such that the existence of heavy quarks may be neglected. Through this approach, we may invoke symmetry to map from the physics of one interaction to another. For example, it may be easy to observe one interaction but hard to observe another which is related to the first via a symmetry. By exploiting this symmetry we may gain insight into behaviours (such as decay rates) of an otherwise hard to measure decay process. The presence of this symmetry is due to the strong interaction not distinguishing electric charge states, within a good approximation.

ii) In 1961 the Ω^- Baryon existence and mass was predicted using SU(3) symmetry by Gell-Mann [1]. The predicted mass of $1672MeV$ was arrived at by invoking the symmetry properties of SU(3) and noting various decay channels possible to provide a lower bound on the mass of the decay reactant. As the Ω^- contains three strange quarks it has a total spin of $\frac{3}{2}$, as was predicted by Gell-Mann.

b) i) Consider the δ^{++} Baryon in its ground state with $l = 0$. The quark constituents are three up quarks. Considering only spin up or spin down states it is then mandatory to have at least two quarks occupying the same quantum state, a contradiction to fermi statistics. This inconsistency is corrected by the introduction of a colour quantum number, assigning each quark either red, green or blue. With this convention the three quarks then have alternate quantum states of differing colour and no longer violate fermi statistics.

ii) As distance between colour charged objects increases the strength of the force between the particles, mediated by gluons, increases in strength. This gives rise to quark confinement which binds quarks and other colour charged objects into colour neutral objects. This confinement makes direct observations of colour charge not possible. This phenomenon is labelled as 'asymptotic freedom'.

iii) The nature of colour charge and the strong interaction permit the creation of gluons around a colour charged object. This process has no limit to the number of gluons surrounding a colour charged object, labelled as a gluon 'dressing'. As gluons themselves carry colour charge, they may also dynamically produce more gluons. This phenomenon by which mass is continually created is labelled 'dynamical mass generation' (DMG). For quarks existing at sufficiently low energies, the mass contributions arising from dynamical mass generation provide a considerable portion of the total mass of the quark system. In the case of colour charged quarks, without considering DMG we refer to the particle as a current quark, the likes of which form Feynman diagram paths. Quarks forming part of the dressing are labelled as constituent quarks and are not conventionally included in Feynman diagrams.

Experimental Evidence for Conservation Laws

Conservation of electric charge

a) Conservation of electric charge. Experiment performed by Moe and Reines (1965). If charge conservation is broken we expect an electron (charge -1) may decay into an electron neutrino (neutral) and a photon. A photon is directed toward an Iodine atom within a NaI crystal. Here the reaction

$$e^- - > \gamma + \nu_e$$

may occur. The produced photon will have high energy ($E_\gamma \ \frac{1}{2}(m_e c^2)$ and will thus be easily visible. The vacant electron energy level may then be occupied by an outer electron, emitted a transitory photon. This photon may be directly observed or may be absorbed by an outer electron which may also be directly observed. The results showed a mean electron lifetime greater than the age of the universe, providing strong support for charge conservation.

Conservation of Baryon number

b) Experiments at Super Kamiokande in Japan attempt to detect the decay of protons into leptons. The decay of a proton into π^0 and e^+ is one such decay channel probed. Decay products are expected to exceed the local speed of light in water, creating cherenkov radiation which may be detected and back-traced to reconstruct the decay dynamics. Results have provided a lower bound on the proton lifetime greater than the current age of universe, yielding support for the conservation of Baryon number and the protons stability.

Conservation of Lepton number

c) Lepton number was originally proposed to be globally conserved such that the total number of leptons and anti-leptons before and after a reaction remained constant. It was later shown that even tighter constraints may be imposed with the conservation of Lepton number $within Lepton families$. Experimental evidence for these two theories are given with Lepton number conservation shown by the absence of processes converting ν_e into $\bar{\nu_e}$ (e) and the absence of processes converting Leptons from the μ family into the electron family and vice-versa (d).

Conservation of Lepton family number

d) A fast beam of μ^- particles is directed towards a concrete slate. This provides the necessary source of Leptons as the beam may decay into e^-, $\bar{\nu_e}$ and $\nu_m u$ particles. The concrete slate absorbs all electrons leaving a narrow neutrino beam of $\bar{\nu_e}$ and $\nu_m u$ particles. This beam is directed toward samples of protons and neutrons. Here, four decays are theorised. Two Lepton family conserved decays

$$\nu_\mu + n \rightarrow \mu^- + p$$
$$\bar{\nu_e} + p \rightarrow e^+ + n$$

and two Lepton family violating decays

$$\nu_\mu + n \rightarrow e^- + p$$
$$\bar{\nu_e} + p \rightarrow \mu^+ + n$$

In experiment the Lepton family violating decay is shown to not be observable, providing confirmation for Lepton family number conservation.

Evidence for $\nu_e \neq \bar{\nu_e}$ Lepton number conservation

e)If lepton number conservation is obeyed, then the decay of a neutron into a proton and electron should be accompanied by an anti-electron neutrino and not by an electron neutrino. If ν_e and $\bar{\nu_e}$ are equivalent then neutrinoless double beta decay should be possible.
This decay channel requires the possibility of conversion of a W Boson into an electron and both electron neutrino or electron anti-neutrinos. Attempts at

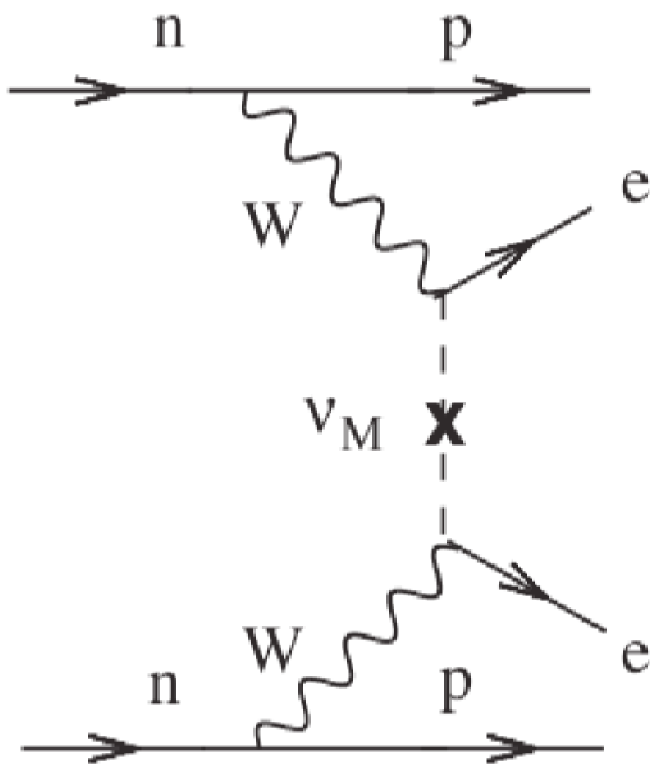

Figure 6:
Proposed neutrinoless double beta decay, the existence of which confirms Lepton number violation.

observation through multiple differing experiments began in the early 2000s, until its occurrence was finally ruled out with the provision of a lower bound on the process timeframe greater than the age of the universe in 2006 [Mod. Phys. Lett. A21 (2006) 1547].