On the

Magnetic Field and other Properties of the Milky Way

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Magnetic field strength varies in location within the Milky Way (Haverkorn 2014) with strong definition along the spiral arms and mild fluctuations due to local environments. The regular magnetic field strength, not including the turbulent fluctuations, is approximately 2 µG (2 x 10^{-10}T) (Haverkorn 2014). Energy density (given the symbol E_ρ hereafter) of the field can be calculated thus:

$$E_\rho = \frac{B^2}{2\mu_0}$$

$$E_\rho = \frac{(2 \times 10^{-10} T)^2}{2(4\pi \times 10^{-7} TmA^{-1})} = 1.592 \times 10^{-14} Jm^{-3}$$

Originating from the Stefen-Boltzmann law it can be shown that energy density for photons emitted at some temperature T is given by the following expression (Hyperphysics 2016):

$$E_\rho = \frac{4}{c}\sigma T^4$$

Where c is the speed of light and σ the Stefan-Boltzmann constant. This formula then yields for the Cosmic Microwave Background:

$$E_\rho = \frac{4}{3 \times 10^8 ms^{-2}}(5.670 \times 10^{-8} Wm^{-2}K^{-4}) \cdot (2.7K)^4 = 4.018 \times 10^{-14} Jm^{-3}$$

Stars with a large mass are capable of undergoing stellar nuclear fusion at a fast rate and as such burn through their mass fuel rapidly. It is for this reason that the most common star type in the galaxy are the lower mass stars, specifically red dwarf stars (Dawson 2017). Typically such stars have a mass of one solar mass and a diameter one 50th that of the sun, where one solar diameter is taken as $1.391 \times 10^6 m$ (Dawson 2017). Using this information, energy density from mass can deduced as follows:

$$m = 2 \times 10^{30} kg$$

$$E = mc^2 = (4 \times 10^{28} kg)(3 \times 10^8 ms^{-2}) = 3.6 \times 10^{45} J$$

Dividing by the volume of an average red dwarf star to represent as an energy density then gives:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{1}{50} \cdot 1.391 \times 10^6 m\right)^3 = 9.019 \times 10^{13} m^3$$

$$E_\rho = \frac{E}{V} = \frac{6 \times 10^{38} J}{9.019 \times 10^{13} m^3} = 3.992 \times 10^{31} Jm^{-3}$$

Given the average thermal energy for per particle in a gas is modelled by the Boltzmann relation:

$$U = \frac{3}{2}KT$$

We can then relate this with the average kinetic energy per particle present in a system and thus derive an expression for velocity as follows:

$$E_k = \frac{1}{2}mv_a{}^2 = \frac{3}{2}KT$$

$$so: v_a = \sqrt{\frac{3KT}{m}}$$

Where v_a is the average velocity per particle particles. For particles at a given temperature in space we see the velocity is then only dependant on the particle mass. Given an electron has a mass considerably less than that of the proton it then follows by the above relation that, on average, electrons travel much faster than protons. In the regions which spacecraft orbit the earth the primary constituents of the environment are ionised hydrogen (protons) and electrons (Rowell 2018). A satellite travelling through this medium will be moving at a speed comparable to that of the protons and so most satellite proton collisions occur on the forward moving face of the satellite. In contrast, fast moving electrons are able to collide rapidly upon all angles and thus induce a negative charge on all sides of the space craft. It is this higher deposition of negative charge in relation to positive which causes a net negative charge to be induced upon a satellite.

A wave passing an object approximately a tenth the size of its wavelength will be susceptible to Rayleigh scattering (Rowell 2018) in which a fraction of the incident waves energy will be deposited into the scattering object. In this scattering regime the power dissipated obeys the following relation (Rowell 2018):

$$P \propto \omega^4$$

Where ω is the angular frequency of the incoming wave. Using this relation we see a fourth power dependence on the frequency resulting in large energy dissipation in the scattering medium.

In the context of seismology, incident seismic waves are often 1-100km in size (Rowell 2018). Since many buildings have dimensions roughly one tenth this scale the buildings are affected by Rayleigh scattering which can have a devastating impact on the structural integrity, especially for high energy seismic waves. In Astrophysical observations of hot stars with atmospheres containing singly ionised Helium, Rayleigh scattering causes scattering of light due to the relative wavelength to particle ratio of approximately less than a tenth of the lights wavelength (Fisak et al. 2016). As such this effect must be taken into consideration when determining the features of atmospheres and other light derived properties of such stars.

Magnetic moment, angular momentum and energy are three quantities which must be conserved in a closed system and it is from this the concept of a magnetic bottle arises. Given angular momentum must be conserved at an orbit radius given by the gyradius r_g we have:

$$r_g = \frac{mv_{pe}}{qB} soL = mv_{pe}r_g = \frac{m^2{v_{pe}}^2}{qB} = constant$$

For a particle of mass m, charge q, within a magnetic field of strength B and velocity perpendicular to that field v_{pe}. From this we see a relationship between v_{pe} and B as:

$$\frac{{v_{pe}}^2}{B} = constant$$

Considering an increase in local magnetic field strength there will then be a corresponding increase in perpendicular velocity by the above relation. Energy conservation then stipulates:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left({v_{pe}}^2 + {v_{pa}}^2\right) = constant$$

Meaning an increase in perpendicular velocity v_{pe} will be met with a decrease in parallel velocity v_{pa} all relative to the direction of the B field. As a magnetic field increase in strength a particle moving within the field will begin to slow in the direction of the field and, provided the particle does not escape the field by some means, will reverse its direction of motion to travel back in the direction it came. This produces a "magnetic bottle" in a region of B field bounded by regions of greater B field strength in which charged particles become trapped. Observable directly on the earth as towards the earth's surface the B field strength increases relative to that in the space surrounding the earth and as such magnetic bottles form – known as the Van-Allen radiation belts.

References

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